Optimal 3GPP Fairness Parameters in 5G NR Unlicensed (NR-U) and WiFi Coexistence

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Abstract—The fifth-generation networks, designed to provide a better quality of service and spectrum utilization, are rapidly being deployed across the world. 3GPP has proposed a fairness criterion, referred to as "3GPP fairness", for the coexistence of 5G New Radio in unlicensed spectrum (NR-U) and WiFi in Release 16. In this correspondence, we derive an analytical expression to determine if the 3GPP fairness is achieved in a given network configuration. We model achieving the 3GPP fairness as an optimization problem and show how to use Sequential Quadratic Programming to find the optimal 5G NR-U parameters from a pre-decided range. We test the optimizer in various conditions and inspect the effect of various parameters. Our results reveal that the optimizer is able to predict the best possible parameters for 3GPP fairness and therefore the proposed method proves useful for tuning 5G NR-U parameters during their coexistence with WiFi.

Index Terms—5G NR-U, WiFi, coexistence, 3GPP fairness, cellular, spectrum sharing, resource management

I. INTRODUCTION

3 RD Generation Partnership Project (3GPP) has standard-ized 5G NR-Unlicensed (NR-U) radio access technology in the Release 16 [1], as the successor of LTE-Licensed Assisted Access (LAA) introduced in Release 13 [2]. 5G NR-U will coexist with other technologies in the unlicensed spectrum, notably WiFi. One of the key concerns to be addressed is setting appropriate ground rules for channel acquisition to ensure quality of service (QoS) for both 5G NR-U and WiFi users. 3GPP has proposed their definition of fairness in Release 16 [1] for 5G NR-U similar to that of LTE-LAA [2], which restricts the WiFi performance to not be reduced by intrusion of NR-U (or LTE-LAA). The LTE-LAA definition has received some criticism in literature because of its one-sided nature, such as in [3]-[4]. Authors in [5] have demonstrated how 3GPP fairness can be achieved in LTE-U and WiFi coexistence by appropriately regulating the duty cycle. Study in [6] has shown that channel access mechanisms can be designed for enabling coexistence where 3GPP fairness is achieved. The work presented in [7] proposed a QoS aware mechanism which has a fairness constraint. It has been shown in [8] that, in at least the common scenarios, 3GPP fairness is satisfied.

Given that 3GPP fairness may not be achieved in every network scenario [4], it would be useful to determine the optimal 5G NR-U parameters such that fairness criterion is met as closely as possible. To the best of our knowledge, limited work has been published for coexistence of 5G NR-U with WiFi [9], and there is no work in literature which proposes a way to tune an arbitrary number of 5G NR-U parameters to achieve 3GPP fairness. A related work [10] focuses on optimizing the network throughput in the coexistence scenario while satisfying 3GPP fairness. The optimization problem in [10] only considers the initial backoff window sizes of 5G NR-U and WiFi as the design variables, which certainly represents a much more limited view than the approach considered in our work. Moreover, our proposed method is extensible to any change in the system model as long as a liberal set of conditions are satisfied, which shall be revealed. Our key contributions can be summarized as follows:

- We characterise the problem of achieving 3GPP fairness as an optimization problem with a non-linear objective function and constraints. We show how the Sequential Quadratic Programming algorithm [11] can be used to solve the problem by tuning 5G NR-U parameters and obtain the "fairest" situation where WiFi 802.11ac and 5G NR-U throughput are the closest possible.
- We inspect the effect of various 5G NR-U parameters on the 3GPP criterion and demonstrate the validity and usefulness of the proposed approach.

II. SYSTEM MODEL

We consider the system model mentioned in [3] and justify how it can be used to model the coexistence of 5G NR-U and WiFi. This system model is an extension of [12] which also serves as a base for [5] and [13]. As per 3GPP Release 16, the coexistence of NR-U and WiFi is enabled by Category-4 LBT, similar to LTE-LAA and WiFi coexistence [1]. The main difference is the variable 5G NR-U slot duration and sub-carrier spacing because of 5G flexible numerology, which introduces an additional degree of freedom and is exploited in this work for a finer tuning and optimization of the coexistence protocol. The same has not been considered in [3]. For the sake of brevity we do not repeat the entire system model. We consider $n^{(W)}$ WiFi nodes, in which $n^{(W)}$ - 1 stations are in association with an access point (AP), and $n^{(L)}$ 5G NR-U nodes, in which $n^{(L)}$ - 1 user equipment are associated with a gNodeB. The nodes of the same type Ψ , where $\Psi \in \{W, L\}$ have identical parameters as summarized in Table I.

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Fig. 1. State transition Markov chain of an individual HOL packet, $\Psi\in\{W,L\},\,N=P^{(\Psi)}+Q^{(\Psi)}-1$

 TABLE I

 System Parameters of WiFi and 5G NR-U

Parameter	Notation
Initial backoff window size	$C^{(\Psi)}$
Maximum backoff stage	$P^{(\Psi)}$
Retry limit	$Q^{(\Psi)}$
Number of sensing slots	$A^{(\Psi)}$
Time duration of a successful transmission	$ au_T^{(\Psi)}$
Time duration of a failed transmission	$ au_F^{(\Psi)}$

We assume that nodes follow an independent timehomogeneous backoff process with an identical steady-state probability of successful transmission of head-of-line (HOL) packets as explained in [12]. The assumption is valid as long as the difference between the number of sensing slots of the nodes is reasonably small. The behaviour of the HOL packets is modeled as a discrete-time Markov renewal process with three categories of states - transmission (*T*), backoff (*R_i*) and collision (*F_i*) where $\forall i \in [0, P^{(\Psi)} + Q^{(\Psi)} - 1]$. The state diagram is as shown in Fig. 1. Though the definition of a slot in 5G NR-U differs from LTE-LAA, we can consider the 5G NR-U slot duration ($D^{(L)}$) instead of the LTE-LAA counterpart when computing the holding times of gNBs [9].

A WiFi node, upon accessing the channel transmits a $PL^{(W)}$ bits payload at $R^{(W)}$ Mbps transmission rate. On the other hand, a 5G NR-U node will transmit for a time duration of maximum channel occupation time (MCOT). We consider some standard assumptions found in literature [3] [4] [12]. The nodes are saturated and the channel is noiseless. i.e., there is no random error. The hidden node problem is assumed to not occur. The WiFi, as well as the NR-U network, carries both uplink and downlink traffic [1]. Also, each node is assumed to have an infinite buffer. Let $\alpha^{(\Psi)}$ be the steady state probability of the channel being idle. For the sake of brevity, we omit some derivations mentioned in [3]. Consider the derived expression of the probability of a HOL packet of type Ψ transmitting:

$$\tilde{\pi}_{T}^{(\Psi)} = \frac{\tau_{T}^{(\Psi)}}{\tau_{T}^{(\Psi)} + \tau_{F}^{(\Psi)} \left(\frac{1-p}{p}\right) + \left(\frac{\sum_{i=0}^{P^{(\Psi)} + Q^{(\Psi)} - 1} (1-p)^{i} \frac{1+C_{i}^{(\Psi)}}{2}}{\alpha^{(\Psi)} \left(1 - (1-p)^{P^{(\Psi)} + Q^{(\Psi)}}\right)}\right)}, \quad (1)$$

2

$$C_i^{(\Psi)} = C^{(\Psi)} 2^{\min\{i, P^{(\Psi)}\}}$$
(2)

 $C_i^{(\Psi)}$ is the backoff window size at the *i*th iteration, $0 \le i \le P^{(\Psi)} + Q^{(\Psi)} - 1$, Let $A_{\min} = \min(A^{(W)}, A^{(L)})$. *p* is the steady state probability of successful transmission of a packet and the solution of the following approximation:

$$p \approx \exp\left\{-\sum_{\Psi \in \{W,L\}} \frac{2n^{(\Psi)} \frac{1 - (1-p)^{P^{(\Psi)} + Q^{(\Psi)}}}{p^{A_{\min} - A^{(\Psi)} + 1}}}{\sum_{i=0}^{P^{(\Psi)} + Q^{(\Psi)}} (1-p)^{i} C_{i}^{(\Psi)}}\right\}$$
(3)

It can be shown that a solution to the above equation exists in the range $0 \le p \le 1$. Expression (3) can be solved using suitable root finding techniques such as classic Brent's method [14]. Let α be the steady state probability of the channel being idle when $A^{(W)} = A^{(L)}$. It is related to its general counterpart by the following expression:

$$\alpha^{(\Psi)} = \alpha p^{A^{(\Psi)} - A_{\min}} \tag{4}$$

We consider the below expression as the starting point of our analysis [3]

$$\begin{aligned} \alpha &= 1 / \left(1 + \tau_{F,\min}(1-p) + \Delta_{\tau_F} \left(1 - \exp\left(\frac{\ln p X^{(\Psi)}}{\Sigma_{\Psi \in \{W,L\}} X^{(\Psi)}}\right) \right) \\ &- \frac{\Sigma_{\Psi \in \{W,L\}} X^{(\Psi)}(\tau_T^{(\Psi)} - \tau_F^{(\Psi)})}{\Sigma_{\Psi \in \{W,L\}} X^{(\Psi)}} p \ln p \right) \quad (5) \end{aligned}$$

Here $\tau_{F,\min} = \min_{\{\Psi \in \{W,L\}\}} \tau_F^{(\Psi)}, \ \psi = \arg\max_{\{\Psi \in \{W,L\}\}} \tau_F^{(\Psi)}, \Delta_{\tau_F} = |\tau_F^{(L)} - \tau_F^{(W)}|$ and

$$X^{(\Psi)} = \frac{\frac{n^{(\Psi)}p^{A^{(\Psi)}}}{C^{(\Psi)}} \left(1 - (1-p)^{P^{(\Psi)} + Q^{(\Psi)}}\right)}{\left(\frac{1}{2p-1} + \left(\frac{1}{p} - \frac{1}{2p-1} - \frac{(1-p)Q^{(\Psi)}}{p}\right)(2-2p)^{P^{(\Psi)}}\right)}$$

III. 3GPP FAIRNESS IN 5G NR-U AND WIFI COEXISTENCE

A. Derivation of the condition for achieving 3GPP fairness

3GPP's definition of fairness [1] states that the NR-U design should target fair coexistence with existing WiFi networks to not impact WiFi services more than an additional WiFi network on the same carrier with respect to throughput and latency. A similar definition was proposed for LTE-LAA [2], for which the authors in [3] and [4] argue, the definition is in favour of WiFi, treating LTE-LAA as an 'intruder' or outsider in the spectrum. We consider the definition for the 5G NR-U case and slightly modify the 'per user throughput' considered in [4]. A similar approach is followed in [5] for coexistence of LTE-U and WiFi. Consider the following scenarios, having the same total number of n nodes:

- Network A, consisting of a WiFi AP and n 1 stations.
- Network B, consisting of x WiFi nodes and n x 5G NR-U nodes $(1 \le x < n)$.

Considering the definition, 3GPP fairness is achieved in the configuration, given x, if the average per-node throughput of a WiFi node in Network B is not lower than the average per-node throughput in Network A [4], [5]. There is no direct condition imposed on 5G NR-U network.

We can now derive an expression to validate if 3GPP fairness is achieved given the network configuration. Node airtime being a measure of throughput can be used in the definition. Let the node fraction-of-time of a node in the network be $\lambda_{out}^{(\Psi)}$ ($\Psi \in \{W, L\}$). In a saturated condition, $\lambda_{out}^{(\Psi)}$ will be equal to its steady state probability of transmission (i.e., service rate):

$$\lambda_{out}^{(\Psi)} = \tilde{\pi}_T^{(\Psi)} \tag{6}$$

Let the total time of observation be T units. We use the suffixes A and B to differentiate between the parameters of the two network models discussed above. For 3GPP fairness,

$$\lambda_{out_A}^{(W)} T \le \lambda_{out_B}^{(W)} T \tag{7}$$

$$\lambda_{out_A}^{(W)} \le \lambda_{out_B}^{(W)} \tag{8}$$

Assuming a saturated network:

$$\tilde{\pi}_{T_A}^{(W)} \le \tilde{\pi}_{T_B}^{(W)} \tag{9}$$

Using (1), we obtain:

$$\begin{aligned} \tau_F^{(W)} \frac{1 - p_A}{p_A} + \left(\frac{\sum_{i=0}^{P^{(W)} + Q^{(W)-1}} (1 - p_A)^i \frac{1 + C_i^{(W)}}{2}}{\alpha^{(W)} (1 - (1 - p_A)^{P^{(W)} + Q^{(W)}})} \right) \\ \ge \tau_F^{(W)} \frac{1 - p_B}{p_B} + \left(\frac{\sum_{i=0}^{P^{(W)} + Q^{(W)-1}} (1 - p_B)^i \frac{1 + C_i^{(W)}}{2}}{\beta (1 - (1 - p_B)^{P^{(W)} + Q^{(W)}})} \right) \quad (10) \end{aligned}$$

The steady-state probability of sensing the channel idle in WiFi only scenario, β can be obtained as [12]:

$$\beta = \frac{1}{1 + \tau_F - \tau_F p - (\tau_T - \tau_F) p \ln p}$$
(11)

3GPP fairness is achieved if the inequality in (10) is satisfied.

B. Achieving 3GPP fairness by tuning 5G NR-U parameters

In this subsection, we are interested in determining if 3GPP fairness can be achieved by tuning a set of 5G NR-U parameters (backoff window size, number of sensing slots, number of nodes, etc.) in a desired range. We now show that achieving 3GPP fairness can be modeled as an optimization problem. From the definition, consider the below function,

$$F(P, p_B) = \min_{P} \left| \lambda_{out_A}^{(W)} - \lambda_{out_B}^{(W)} \right|$$
(12)

where $P = \{P_i \mid i \in [1, l]\}$ denotes a vector of 5G NR-U parameters considered for tuning, which can be one or more from Table I, along with the 5G NR-U slot duration. Notice that, according to the 3GPP definition, fairness is achieved when the condition in (8) is met, with the equality representing the limit condition at which fairness is still achieved. Therefore, minimizing the difference between the two terms in (8) as represented by (12) ensures that fairness is approached as closely as possible (i.e., as allowed by the particular set of operating conditions) in a worst-case scenario.

 p_B , which is constrained by (3), must at the same time satisfy the inequality $0 \le p_B \le 1$. Ideally, 3GPP fairness is achieved when F = 0, as per (8). For all practical purposes, it is reasonable to argue that 3GPP fairness is achieved when the optimized value is less than a predefined small threshold ϵ .

We define the following functions for modeling our problem. Consider,

$$G(P, p_B) = p_B - \exp\left\{-\sum_{\Psi \in \{W, L\}} \frac{2n^{(\Psi)} \frac{1 - (1 - p_B)^{P^{(1)} + Q^{(1)}}}{p_B^{A_{min} - A^{(\Psi)} + 1}}}{\sum_{i=0}^{P^{(\Psi)} + Q^{(\Psi)}} (1 - p_B)^i C_i^{(\Psi)}}\right\}$$
(13)

$$q_0(p_B) = p_B \tag{14}$$

$$q_1(p_B) = 1 - p_B \tag{15}$$

Our problem can, therefore, be expressed as:

minimize
$$F(P, p_B)$$

subject to $q_0(p_B) \ge 0$
 $q_1(p_B) \ge 0$ (16)
 $G(P, p_B) = 0$
 $g_i(P) \ge 0 \ \forall i \in [1, t]$

Constraint (13) is a consequence of (3). Here, $g_i(P)$ denotes the set of functions (distinguished by *i*) which define the bounds on the values of the parameters considered, analogous to (14) and (15). The Lagrangian function for this problem can be expressed as

$$\mathcal{L}(P, p_B, \lambda, \mu, \delta, \sigma_0 \cdots \sigma_k) = F(P, p_B) - \lambda^T q_0(p_B) - \mu^T q_1(p_B) - \delta^T G(P)$$
(17)
$$- \Sigma_{i=0}^t \sigma_i^T g_i(P)$$

The optimization equation (16) expresses our 3GPP fairness objective function while taking into consideration constraints from the analytical model as well as the parameters. Given the continuous nature of the expressions, (16) can be solved using the Sequential Quadratic Programming (SQP) method [11], a class of algorithms for solving nonlinear optimization problems. Let X_k denote F's parameters at the kth iteration. Similarly, let the subscript k in a function denote its value at the corresponding iteration. SQP determines the direction of search d_k as the solution of the subproblem [11]:

minimize
$$F_k + \nabla F_k^T d + \frac{1}{2} d^T \nabla_{XX}^2 \mathcal{L}_k d$$

subject to $\nabla G(P_k, p_B)^T d + G(P_k, p_B) = 0$
 $\nabla q_0(p_{B_k})^T d + q_0(p_{B_k}) \ge 0$
 $\nabla q_1(p_{B_k})^T d + q_1(p_{B_k}) \ge 0$
 $\nabla g_i(p_{B_k})^T d + g_i(p_{B_k}) \ge 0 \quad \forall i \in [1, t]$

$$(18)$$

The above expression is a quadratic programming problem and can be solved using standard techniques [11]. The SQP algorithm for our context is described as Algorithm 1. In a static network configuration, the parameters can be computed in advance. In the contrary case, the gNB should obtain the WiFi parameters from a central authority or through crosstechnology communication. An approach for the latter for LTE-U and WiFi coexistence has been proposed in [15],



Fig. 2. Comparison of node airtime in networks A and B on varying $n^{(L)}$ with different values of $A^{(L)}$. $(n^{(W)} = 20)$



Fig. 4. Comparison of node airtime in networks A and B on varying $A^{(L)}$.

Algorithm 1 Determine best 5G NR-U Parameters for Achieving 3GPP Fairness

1:	procedure SQP-3GPPFAIRNESS
2:	while a convergence test is satisfied do
3:	Evaluate F_k , ∇F_k , $\nabla^2_{XX} \mathcal{L}_k$, G_k ,
4:	$q_{0_k}, q_{1_k}, g_{1_k}, g_{2_k} \cdots g_{t_k}$
5:	Solve (18) to obtain d_k , μ_k , δ_k , σ_{1_k} , $\sigma_{2_k} \cdots \sigma_{t_k}$
6:	$X_{k+1} \Leftarrow X_k + d_k$
7:	end while
8:	end procedure

discussion of which is beyond the scope of this work. Existing implementations can be referred to for an efficient incorporation in a practical system, for example a software package for SQP described in [16].

IV. NUMERICAL RESULTS

We validate numerically our analysis, and the set of parameters generated by Algorithm 1. The results involving one varying parameter have been verified using Monte Carlo simulations in MATLAB, which are represented as dots in the subsequent figures. We execute each experiment for 10^8 time



Fig. 3. Comparison of node airtime in networks A and B on varying $n^{(L)}$ with different values of $C^{(L)}$. $(n^{(W)} = 20)$



Fig. 5. Comparison of node airtime in networks A and B on varying $C^{(L)}$.

slots, averaging each result 11 times for accurate results. The parameters are summarized in Table II. Packet arrival in each of the nodes is simulated as a Bernoulli process in each node with a fixed rate $\lambda^{(\Psi)}$ per type of node per iteration. $\lambda^{(\Psi)}$ is uniformly distributed within [0,1). We use SciPy's optimization tools [17] which provide an implementation of Sequential Least Squares Programming, where (18) is substituted by an equivalent linear least squares subproblem [16].

In each of the results, we compare the node airtime values of Networks A and B. When 3GPP fairness is not achievable, Algorithm 1 will generate the best possible parameters along with the difference in throughput as compared to the ideal case.

Our results will validate the feasibility of 3GPP fairness by changing various parameters. We use Algorithm 1 from the perspective of a designer - tweaking parameters one at a time until 3GPP fairness is achieved. Figs. 2-3 involve varying $n^{(L)}$ on the X-axis keeping $n^{(W)} = 20$. Fig. 2 compares the node airtime values by tuning $A^{(L)}$ and $C^{(L)}$. The difference between throughput is significant when we consider the default setting. We first introduce $A^{(L)}$ as a tuning parameter such that $A^{(L)} \in [2, 8]$. The WiFi throughput improves as we expect but the optimizer cannot find a set of parameters to achieve 3GPP fairness. Introducing $C^{(L)} \in [8, 64]$ as a parameter causes a

20 µs
112 bits+PHYH
34 µs
16 µs
9 μs
6 Mbps
2 ¹⁶ bits
54 Mbps
16
2
6
1
6
1
8 <i>ms</i>
500 µs

TABLE II Simulation Parameters

marginal improvement in the throughput. It can be noticed that the simulation results can slightly deviate from analytical results in some cases. This is particularly prominent when $A^{(L)}$ is significantly larger than $A^{(W)}$. The time-homogeneity assumption of the model holds good when this difference is not very large. If not, the analytical model underestimates $\lambda_{out}^{(W)}$ and hence the difference. [3] Nevertheless, it is worth noting that, despite the slight difference observed in such particular case, both analytical and simulation results show an overall agreement, which confirms the correctness and validity of our analytical results. Fig. 3 considers tuning $C^{(L)}$ in range [8, 128]. We then consider introducing $PL^{(W)}$ as a parameter in range $[2^{16}, 2^{24}]$. The optimizer fetches a point where 3GPP fairness can be achieved - $n^{(L)} = 5, C^{(L)} = 128, PL^{(W)} = 2^{20}$. 802.11ac standards allow a maximum payload size of 2^{24} bits as a consequence of frame aggregation. [18] Because of the wide range of $PL^{(W)}$, it can be tweaked as a parameter to achieve 3GPP fairness.

Figs. 4-5 consider a constant number of nodes in the network such that $n^{(W)} + n^{(L)} = 30$. In Fig. 4 we consider $A^{(L)}$ as our independent variable. The results are significantly far from 3GPP fairness even when the number of 5G NR-U nodes are increased to as many as 50% of total. Reducing the NR-U slot duration to $D^{(L)} = 62.5 \mu s$ which is the lowest possible causes some improvement in the throughput. Introducing $PL^{(W)}$ as a parameter in range [2¹⁶, 2²⁴] achieves 3GPP fairness around $A^{(L)} = 6$. This illustration also highlights the usefulness of the technique in obtaining a feasible solution if the number of permutations of the design variables is large, in this case, given the large range of $PL^{(W)}$, resulting in $O(10^8)$ permutations overall. Obtaining parameters numerically, in this case, would have been infeasible. Fig. 5 deals with the variation against $C^{(L)}$. 3GPP fairness is achieved at around the point $C^{(L)} = 16$ for the different cases shown in the figure similar to Fig. 4.

V. CONCLUSION

In this correspondence, we have formulated the problem of achieving 3GPP fairness as an optimization problem. We proposed how to use Sequential Quadratic Programming to tune 5

5G NR-U parameters to achieve fairness. The results show that the proposed method can be effectively used to predict the most suitable 5G NR-U parameters which are closest to satisfying the 3GPP criterion. It is also revealed that 3GPP fairness need not be achievable in general, and often when it is, either the 5G NR-U parameters are significantly larger than typical WiFi parameters, which would imply a reduced throughput, the number of 5G NR-U nodes are significantly lower than WiFi, or the payload size of WiFi is increased. The proposed framework has proven useful in determining a suitable configuration for a fair coexistence of 5G NR-U and WiFi networks.

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