Energy Detection Based Estimation of Primary Channel Occupancy Rate in Cognitive Radio

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Abstract—Dynamic Spectrum Access (DSA)/Cognitive Radio (CR) systems can detect transmission opportunities by means of periodic spectrum sensing. The design and configuration of spectrum sensing is commonly aimed at optimising the instantaneous detection of such opportunities. Besides the detection of transmission opportunities, spectrum sensing can also be exploited to provide DSA/CR systems with more sophisticated and elaborated information, including for instance statistical information on the occupancy pattern of primary channels. However, the configuration of spectrum sensing in order to minimise the estimation error of channel activity statistics has received much less attention. In this context, this work explores the configuration of an energy detector in order to enable an accurate estimation of the real occupancy rate of a primary channel, thus providing DSA/CR systems with accurate statistical information of primary channels that can be used effectively in spectrum and radio resource management decisions.

Keywords—Cognitive radio, dynamic spectrum access, primary activity statistics, channel occupancy rate, energy detection.

I. INTRODUCTION

The Dynamic Spectrum Access (DSA) principle [1], based on the Cognitive Radio (CR) paradigm [2], aims at increasing spectrum efficiency by allowing unlicensed (secondary) users to opportunistically access licensed bands temporarily/spatially unused by the licensed (primary) users.

One of the most important requirements for a DSA/CR system is not to cause harmful interference to primary users. In order to guarantee interference-free spectrum access, secondary users can monitor the instantaneous busy/idle occupancy state of primary channels by means of periodic sensing. Spectrum sensing methods make a binary decision on the busy/idle state of a channel based on a set of signal samples of the channel. The main purpose of spectrum sensing is to determine whether a signal is present in a primary channel so that the DSA/CR system can decide whether the channel can be accessed opportunistically or needs to be vacated after the return of a primary user to the channel. However, besides the detection of transmission opportunities, spectrum sensing can also be exploited to provide the DSA/CR system with more elaborated information. The sequence of binary busy/idle decisions for a channel can be further processed in order to compute relevant channel activity statistics such as the duration of busy/idle periods, their minimum, mean and variance, or the underlying distribution [3]. The knowledge of primary activity statistics can be exploited by DSA/CR systems in several ways, including for instance the prediction of future trends in the spectrum occupancy [4], the selection of the most appropriate channel/band of operation [5], and other spectrum and radio resource management decisions to optimise the system performance and spectrum efficiency [6].

The design and configuration of spectrum sensing algorithms is commonly aimed at optimising the instantaneous detection of spectrum opportunities. However, the configuration of spectrum sensing algorithms in order to maximise the accuracy of estimation of channel activity statistics constitutes an area that has received much less attention [7]. In this context, this work explores the configuration of spectrum sensing for the estimation of channel activity statistics.

A wide range of sensing methods has been proposed in the context of DSA/CR [8]. Despite its practical performance limitations, energy detection [9] has gained popularity as a spectrum sensing technique due to its general applicability and simplicity as well as its low computational and implementation costs. Energy detection has been a preferred approach for many past spectrum sensing studies and also constitutes the spectrum sensing method considered in this work. Energy detection compares the energy received in a channel to a properly set decision threshold. If the energy lies above (below) the threshold, the channel is declared to be busy (idle). The selection of the decision threshold constitutes a key aspect in the configuration of an energy detector. While existing methods are aimed at optimising the instantaneous detection of spectrum opportunities, this work explores a novel threshold selection criterion aimed at minimising the estimation error of the Channel Occupancy Rate (COR), which represents the probability (theoretical definition) or fraction of time (empirical definition) that a channel is occupied by a primary user. The interest of considering the COR as representative channel activity statistic relies on its ability to quantify in a simple way the amount of spectrum opportunities that a DSA/CR system can expect to find in a primary channel. A DSA/CR system can estimate the COR for several primary channels and then select the channel with the lowest COR (i.e., the channel with highest expected amount of transmission of opportunities). Since the observed COR is computed based on spectrum sensing observations, it may differ from the actual COR due to sensing errors. The objective of this work is to investigate the configuration of the energy detection method (i.e., the selection of the decision threshold) in order to minimise the error in the estimated COR, thus providing DSA/CR systems with an accurate estimation of the real COR that can be used effectively in spectrum and radio resource management decisions.
The rest of this work is organised as follows. First, Section II provides an overview of spectrum sensing based on energy detection. Then, Section III reviews existing threshold-selection methods for energy detection and Section IV proposes a novel method aimed at minimising the COR estimation error. The performance of the considered methods is assessed by means of simulations, following the simulation approach presented in Section V, and their accuracy in estimating the COR is compared and analysed in Section VI. Finally, Section VII summarises and concludes the paper.

II. ENERGY DETECTION

A. Operating principle

Energy detection measures the energy of the samples $y[n]$ (which include pure primary signal and noise) taken from a primary channel during an observation interval of $N$ samples and declares the channel state as busy (hypothesis $H_1$) if the measured energy is greater than a decision threshold $\lambda$, or idle (hypothesis $H_0$) otherwise [9]:

$$\sum_{n=1}^{N} |y[n]|^2 \geq \frac{\lambda}{\lambda_0}$$  \hspace{1cm} (1)

B. Theoretical performance

An ideal spectrum sensor would select hypothesis $H_1$ whenever a primary signal is present and hypothesis $H_0$ otherwise. In practice, spectrum sensing algorithms are affected by errors, which can be classified into missed detections (when a primary signal is present in the sensed channel and the spectrum sensing algorithm selects hypothesis $H_0$) or false alarms (when the sensed spectrum channel is idle and the spectrum sensing algorithm selects hypothesis $H_1$). The performance of any spectrum sensing algorithm can be summarised by means of two probabilities: the probability of missed detection $P_{md} = P(H_0|H_1)$, or its complementary probability of detection $P_d = P(H_1|H_1) = 1 - P_{md}$, and the probability of false alarm $P_{fa} = P(H_1|H_0)$. For an energy detector, these probabilities can be approximated by [10]:

$$P_d(\lambda) = Q\left(\frac{\lambda - N(\lambda_0^2 + \sigma_w^2)}{\sqrt{2N}\sigma_w}\right)$$ \hspace{1cm} (2)

$$P_{fa}(\lambda) = Q\left(\frac{\lambda - N\sigma_w^2}{\sqrt{2N}\sigma_w^2}\right)$$ \hspace{1cm} (3)

where $Q(\cdot)$ is the Gaussian tail probability $Q$-function [11, (26.2.3)], $\sigma_p^2$ is the received primary signal power and $\sigma_w^2$ is the noise power. As it can be appreciated, the performance of energy detection depends on the selected decision threshold $\lambda$, which constitutes a key design parameter.

III. EXISTING THRESHOLD-SELECTION METHODS

This section reviews several threshold-selection criteria proposed in the literature, which have as a common objective the maximisation of detected spectrum opportunities. While this objective is different from the one pursued in this work, these criteria are considered as a reference due to the lack of methods specifically envisaged for COR estimation.

A. Constant False Alarm Rate (CFAR)

A common approach in the existing literature is to select the decision threshold so that a certain target probability of false alarm $P_{fa}^*$ is met [12]. Solving (3) for $\lambda$ yields the optimum threshold $\lambda^*$ according to the CFAR criterion:

$$\lambda^* = \left(\frac{1}{N} P_{fa}^* \sqrt{2N} + N\right) \frac{\sigma_w^2}{\sigma_w^2}$$ \hspace{1cm} (4)

which only requires the noise power $\sigma_w^2$ to be known.

B. Constant Signal Detection Rate (CSDR)

An alternative approach is to select the decision threshold so that a certain target probability of detection $P_d^*$ is met [13]. Solving (2) for $\lambda$ yields the optimum decision threshold $\lambda^*$:

$$\lambda^* = \left(\frac{1}{N} P_d^* \sqrt{2N} + N\right) \frac{\sigma_w^2}{\sigma_w^2} + 1$$ \hspace{1cm} (5)

which requires not only the noise power $\sigma_w^2$ but also the Signal-to-Noise Ratio (SNR) $\gamma = \sigma_p^2/\sigma_w^2$ to be known.

C. Minimum Sensing Error Rate (MSER)

Another option is to select the decision threshold so that the total sensing error rate is minimised [12, 14]. To this end, a total sensing error function is defined as:

$$f_e(\lambda) = P_{fa}(\lambda) + P_{md}(\lambda)$$ \hspace{1cm} (6)

and the optimum threshold is then obtained as:

$$\lambda^* = \arg \min_{\lambda} f_e(\lambda)$$ \hspace{1cm} (7)

which can be calculated by solving $df_e(\lambda)/d\lambda = 0$ for $\lambda$:

$$\lambda^* = \left(1 + \frac{4}{N} \gamma + \frac{2}{N} \ln(\gamma + 1) \right) \frac{\gamma + 1}{\gamma + 2} N\sigma_w^2$$ \hspace{1cm} (8)

It can be shown that $d^2 f_e(\lambda)/d\lambda^2|_{\lambda=\lambda^*} > 0$. Therefore, $f_e(\lambda)$ has a global minimum at $\lambda = \lambda^*$. Note that (8) requires both the noise power $\sigma_w^2$ and the SNR $\gamma$ to be known.

IV. PROPOSED THRESHOLD-SELECTION METHOD

A. Accurate Channel Occupancy Rate (ACOR)

The objective of the proposed criterion is not to maximise the detection of spectrum opportunities by minimising sensing error rates, but minimise the COR estimation error.

Let $\Psi$ be the real COR of a primary channel and $\hat{\Psi}$ the COR estimated by a DSA/CR user (based on spectrum sensing observations). A primary channel is observed as busy when it is busy and successfully detected as such, or when it is idle but observed as busy because of a false alarm. Hence:

$$\hat{\Psi} = P(H_1)P(H_1|H_1) + P(H_0)P(H_1|H_0)$$

$$= \Psi P_d(\lambda) + (1 - \Psi) P_{fa}(\lambda)$$ \hspace{1cm} (9)

The objective is to minimise the COR estimation error:

$$f_e(\lambda) = |\Psi - \hat{\Psi}|$$ \hspace{1cm} (10)

Hence, the optimum decision threshold is given by:

$$\lambda^* = \arg \min_{\lambda} f_e(\lambda)$$ \hspace{1cm} (11)
Figure 1 shows an example of the error function in (10) for various COR ($\Psi$) and SNR ($\gamma$) values. As it can be appreciated, $\lambda^*$ is sometimes achieved at values of $\lambda$ for which $f_e(\lambda)$ is not differentiable. Therefore, $\lambda^*$ cannot be calculated by solving $df_e(\lambda)/d\lambda = 0$ for $\lambda$. However, Figure 1 indicates that the minimum estimation error that can be attained is zero. Therefore, $\lambda^*$ can be obtained by solving $f_e(\lambda) = |\Psi - \hat{\Psi}| = 0$ for $\lambda$, which leads to the expression:

$$
\frac{P_{fa}(\lambda^*)}{1 - P_d(\lambda^*) + P_{fa}(\lambda^*)} = \Psi
$$

This result indicates that an exact estimation of the actual COR of a primary channel is feasible if the decision threshold is selected in such a way that the resulting probabilities of detection and false alarm, according to (2) and (3), satisfy the relation expressed in (12). While (12) cannot be solved in closed-form for $\lambda$, the optimum decision threshold $\lambda^*$ can readily be obtained from (12) by means of simple numerical methods. However, this requires not only the SNR and noise power ($\gamma$ and $\sigma^2$) to be known but also the exact COR value itself ($\Psi$), which leads to a *chicken-and-egg problem*. To solve this issue, an iterative threshold-setting algorithm is proposed.

### B. Iterative threshold-setting algorithm

According to (12), setting the optimum decision threshold that enables an accurate estimation of the real COR requires the knowledge of the COR itself. However, the COR of a primary channel is unknown and indeed the parameter to be estimated. Based on the result of (12), the iterative process of Algorithm 1 is proposed, which is aimed at iteratively adapting the decision threshold until the right value is reached and an accurate estimation of the COR is obtained.

The algorithm needs as input information an initial guess of the COR ($\hat{\Psi}_0$), the SNR of the primary channel ($\gamma$) which can be estimated by using appropriate SNR estimation methods, and the selected sensing period ($N$). The output produced by the algorithm is the estimated COR ($\hat{\Psi}$). In the first iteration of the algorithm, a decision threshold $\lambda^*$ is computed by numerically solving (12) for $\lambda$, assuming that the COR is the initial guess $\hat{\Psi}_0$. Based on an energy detector with the computed decision threshold $\lambda^*$, the sequence of energy values measured in the primary channel is converted to a sequence of busy/idle states, which is used to estimate the COR ($\hat{\Psi}$). The estimated $\Psi$ is then used to recompute a new decision threshold $\lambda^*$, which is applied to the same sequence of measured energy values in order to obtain a new (and eventually more accurate) estimation of the COR ($\hat{\Psi}$). In each new iteration, the value of the employed decision threshold $\lambda^*$ is recomputed based on the latest COR estimation $\hat{\Psi}$. The underlying hypothesis of this algorithm is that after a sufficiently high number of iterations the algorithm should reach the desired decision threshold so that the real COR should be estimated accurately.

### Algorithm 1 Iterative threshold-setting algorithm

**Input:** $\Psi_0 \in (0,1)$, $\gamma \in \mathbb{R}^+$, $N \in \mathbb{N}$

**Output:** $\hat{\Psi} \in (0,1)$

1: $\chi \leftarrow \hat{\Psi}_0$

2: $\lambda^* \leftarrow \text{Numerical solution for } \lambda \text{ of } \frac{P_{fa}(\lambda)}{1 - P_d(\lambda) + P_{fa}(\lambda)} = \chi$

3: $\hat{\Psi} \leftarrow \text{COR estimation based on decision threshold } \lambda^*$

4: $\chi \leftarrow \hat{\Psi}$

5: Go to step 2

The performance of the threshold-selection methods considered in this work is evaluated by means of simulations. The employed simulation approach is based on the following steps:

1) Generate a random binary sequence of channel states with a predefined COR value. This sequence represents the real busy/idle states of a primary channel and is generated in such a way that the quotient between the number of busy samples and total number of samples meets a predefined COR value $\Psi$.

2) Determine the binary sequence of channel states that would be observed by a DSA/CR user after sensing the sequence generated in step 1:

   a) Compute the optimum decision threshold $\lambda^*$ according to the considered criterion, based on (4) for CFAR, (5) for CSDR, (8) for MSER, and (12) for ACOR.

   b) Compute the probabilities of detection and false alarm corresponding to the decision threshold obtained in step 2.a, $P_d(\lambda^*)$ and $P_{fa}(\lambda^*)$, based on (2) and (3), respectively.

   c) Process each sample (channel state) of the binary sequence generated in step 1 as follows: when the channel is idle, the observed state may randomly change to busy with probability $P_{fa}(\lambda^*)$; when the channel is busy, the observed state may randomly change to idle with probability $P_{rid}(\lambda^*) = 1 - P_d(\lambda^*)$. Notice that random sensing errors are introduced in the sequence generated in step 1 according to the error probabilities corresponding to the selected decision threshold $\lambda^*$.

3) Compute the COR estimated by a DSA/CR user $\hat{\Psi}$ as the COR value of the sequence obtained in step 2.
Therefore, CF AR COR estimations at high SNR values are more accurate for lower target $P_{fa}^*$ values. In the case of CSDR, the provision of a constant signal detection rate implies necessarily the occurrence of signal misdetections, even at high SNR values. As a result, the channel is observed as busy less often than it actually is and thus CSDR leads to an underestimation of the COR. Thus, CSDR COR estimations at high SNR values are more accurate for higher target $P_{fa}^*$ values. The accuracy of both CF AR and CSDR degrades significantly as the SNR decreases. It is interesting to note that the more accurate these methods are at high SNR values, the more significant the accuracy degradation is at low SNR values.

Compared to CF AR and CSDR, MSER constitutes a more convenient approach for COR estimation. The balance between false alarm and signal detection rates provided by this method results in a more accurate COR estimation at high SNR values. However, the accuracy of the MSER COR estimation degrades at low SNR values as it is also the case of CF AR and CSDR. As it can be appreciated, the accuracy of these three methods begins to experience a significant degradation as the SNR decreases below a certain threshold (0 dB in these examples).
On the other hand, the COR estimated by the proposed ACOR method shows a remarkable accuracy for the whole range of SNR values as shown in Figure 5. The threshold-selection criterion of ACOR is capable to balance the false alarm and signal detection rates, not only at high but also low SNR values, thus providing an accurate COR estimation at any SNR. The only difference between low and high SNR conditions is the number of iterations required by ACOR to converge to the right decision threshold and provide a sufficiently accurate estimation, which is illustrated in Figure 6 (the initial COR guess for this example is $\Psi_0 = 0.5$). As it can be appreciated, the number of iterations required is higher for low SNR values. At high SNR values, the specific value of the decision threshold does not play a crucial role. The reason is that for high SNR values there is a wide range of valid decision thresholds between the maximum noise power and the minimum signal power. Any decision threshold within this range will eventually lead to a perfect spectrum sensing performance (i.e., $P_a = 1$ and $P_{fa} = 0$) and thus an accurate COR estimation (see Figure 1). As a result, the specific value of the decision threshold is not crucial at high SNR values. As shown in Figure 6 for an SNR of 5 dB (i.e., a relatively high SNR value), the COR is accurately estimated in the first iteration of ACOR, for all COR values, despite having chosen an arbitrary initial guess ($\Psi_0 = 0.5$). The range of valid decision thresholds becomes narrower as the SNR decreases. For low SNR values, where signal and noise power levels can be similar, the specific value of the decision threshold is crucial for an accurate COR estimation (see Figure 1). In such a case, the initial guess $\Psi_0$ is in general inadequate and several iterations of ACOR are required in order to find the right decision threshold. The example of Figure 6 indicates that the number of iterations required to estimate the real COR with a maximum absolute error of 0.01 at an SNR of –15 dB (i.e., a relatively low SNR value) is 68 (for $\Psi = 0.05$), 42 (for $\Psi = 0.2$), 1 (for $\Psi = 0.5$), 48 (for $\Psi = 0.8$) and 97 (for $\Psi = 0.95$). The number of iterations required by ACOR to provide a COR estimation with a certain degree of accuracy depends on the real COR, the SNR and the initial COR guess $\Psi_0$ of the iterative threshold-setting algorithm. However, the results shown in Figure 6 demonstrate that ACOR is capable to converge to an arbitrarily accurate estimation of the real COR regardless of the specific operation conditions. This makes of ACOR an excellent method to accurately estimate the real COR of unknown primary channels in DSA/CR systems.

B. Impact of imperfect SNR estimation

The proposed ACOR method is able to estimate accurately the COR of a primary channel at any arbitrary SNR, provided that the SNR is known. The knowledge of the SNR is required in order to set the decision threshold based on the numerical resolution of (12). This section analyses the impact of an imperfect SNR estimation on the ACOR accuracy with respect to CSDR and MSER. Notice that the CFAR decision threshold in (4) is independent of the SNR and therefore CFAR is not considered in the study carried out in this section.

The obtained simulation results indicated that the COR estimation error depends not only on the SNR error but also on the SNR value itself. For an SNR of 5 dB (high SNR), it was observed that the COR estimated by CSDR, MSER and ACOR is not affected by the SNR error (the estimated COR was observed to be the same as for a perfect SNR estimation even for high SNR errors of ±6 dB). As explained in Section VI-A, for high SNR values there is a wide range of valid decision threshold values for which the sensing performance is perfect. As a result, the specific value of the decision threshold is not crucial under high SNR conditions. An error in the estimated SNR will lead to an incorrect decision threshold. However, the resulting decision threshold may still lie within the range of perfect sensing performance. As a result, the COR estimated by CSDR, MSER and ACOR is not affected significantly by SNR estimation errors if the COR is sufficiently high. However, the SNR error may have an important impact under low SNR conditions. Figures 7, 8 and 9 show the COR estimation error for CSDR (with $P_t = 0.90$), MSER and ACOR, respectively, for an SNR of –15 dB (i.e., a low SNR value). As it can be observed, the COR estimation error for CSDR and MSER is not significantly affected by the SNR error. This is due to the fact that CSDR and MSER are characterised by a high COR estimation error at low SNR values, even for a perfect SNR estimation (see Figures 3 and 4). As a result, an SNR error does not contribute significantly to the already high COR estimation error of these methods. Nevertheless, in the case of ACOR, which is able to provide an accurate COR estimation under perfect SNR estimation (i.e., SNR error of 0 dB in Figure 9), the COR estimation error increases rapidly as the SNR error increases. This indicates that ACOR is sensitive to SNR estimation errors. In particular, Figure 9 indicates that SNR errors of 2 dB or more may lead to appreciable COR estimation errors. Therefore, an accurate estimation of the COR based on the ACOR method requires accurate SNR estimation methods.

The accuracy of ACOR depends on the accuracy of the underlying SNR estimation method. Under sufficiently accurate SNR estimations, ACOR is able to provide nearly perfect estimations of the real COR of unknown primary channels. It is worth noting that, despite its sensitivity to SNR errors, ACOR is significantly more accurate than CFAR, CSDR and MSER, even in the presence of SNR errors. Therefore, under realistic conditions ACOR still constitutes a more convenient COR estimation method compared to other existing alternatives.
to be significantly more accurate than other existing threshold-conditions (i.e., real COR and SNR). ACOR has been shown to identify transmission opportunities in primary channels regardless of the specific operation and configuration of spectrum sensing algorithms has traditionally been aimed at optimising the instantaneous detection of primary channel activity statistics, which can be exploited in DSA/CR systems reliance on spectrum sensing observations to identify transmission opportunities in primary channels. Besides the detection of transmission opportunities, spectrum sensing observations can also be exploited to provide the DSA/CR system with more elaborated information (e.g., primary channel activity statistics), which can be exploited in spectrum and radio resource management decisions. The design and configuration of spectrum sensing algorithms has traditionally been aimed at optimising the instantaneous detection of spectrum opportunities. However, the estimation of channel activity statistics based on spectrum sensing has received less attention. This work has analysed the configuration of an energy detector (i.e., the selection of the decision threshold) in order to minimise the COR estimation error. The obtained results demonstrate that the proposed ACOR method is able to provide an arbitrarily accurate estimation of the real COR of an unknown primary channel regardless of the specific operation conditions (i.e., real COR and SNR). ACOR has been shown to be significantly more accurate than other existing threshold-

**VII. CONCLUSIONS**

DSA/CR systems rely on spectrum sensing observations to identify transmission opportunities in primary channels. Besides the detection of transmission opportunities, spectrum sensing observations can also be exploited to provide the DSA/CR system with more elaborated information (e.g., primary channel activity statistics), which can be exploited in spectrum and radio resource management decisions. The design and configuration of spectrum sensing algorithms has traditionally been aimed at optimising the instantaneous detection of spectrum opportunities. However, the estimation of channel activity statistics based on spectrum sensing has received less attention. This work has analysed the configuration of an energy detector (i.e., the selection of the decision threshold) in order to minimise the COR estimation error. The obtained results demonstrate that the proposed ACOR method is able to provide an arbitrarily accurate estimation of the real COR of an unknown primary channel regardless of the specific operation conditions (i.e., real COR and SNR). ACOR has been shown to be significantly more accurate than other existing threshold-

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